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Application of Koob's Analysis
to a Shaft Governor

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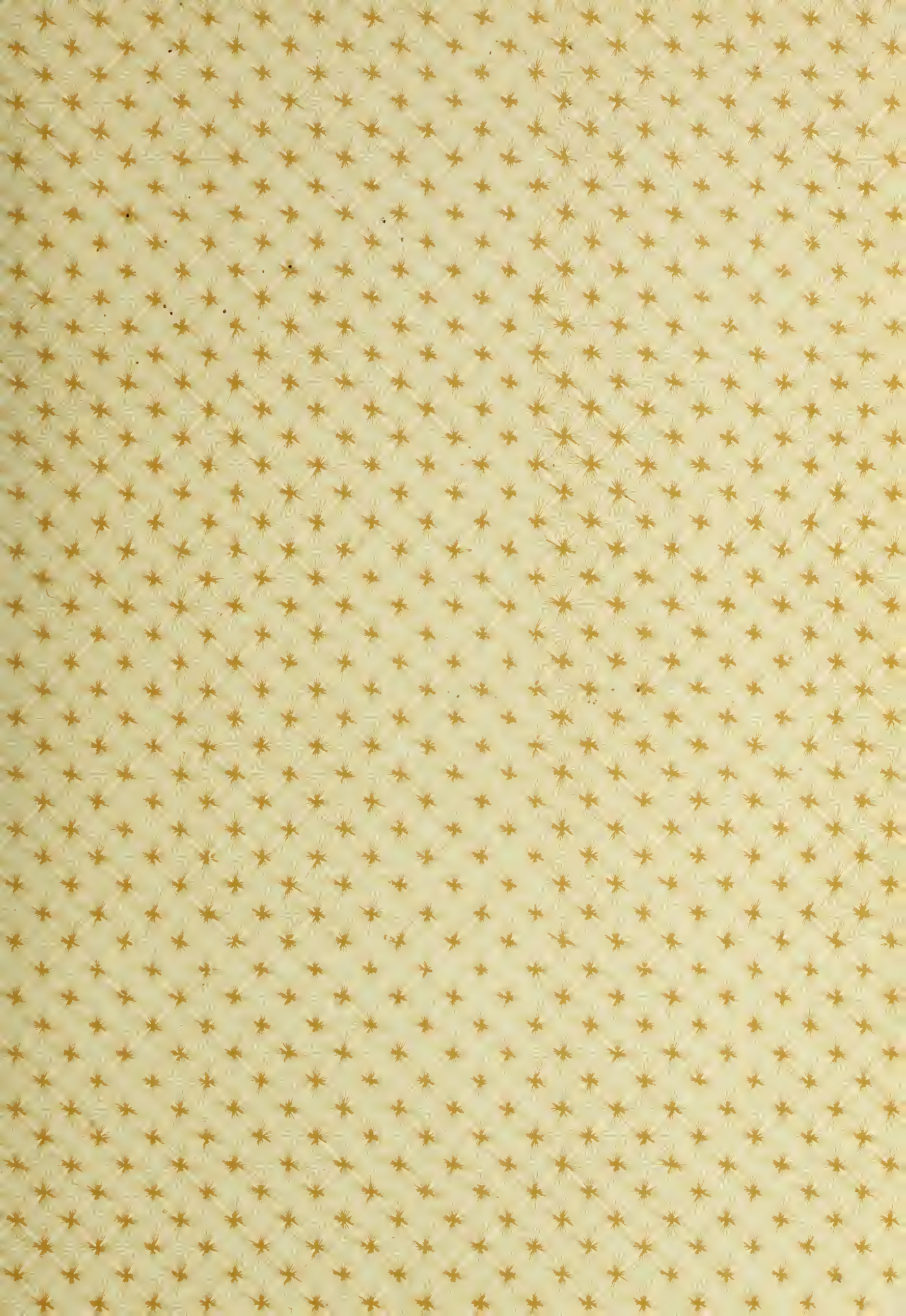
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APPLICATION OF KOOB'S ANALYSIS TO A SHAFT GOVERNOR

BY

Willard Lacy Pollard

THESIS FOR THE DEGREE OF BACHELOR OF SCIENCE
IN MECHANICAL ENGINEERING

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WILLARD LACY POLLARD

ENTITLED.....APPLICATION OF KOOB'S ANALYSIS TO A SHAFT GOVERNOR.....

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE
OF Bachelor of Science in Mechanical Engineering

L. P. Brackenridge

HEAD OF DEPARTMENT OF Mechanical Engineering

C O N T E N T S .

- I Introduction- Statement of the Problem.
- II Koob's Graphical Solution of the Governor Problem.
- III Determination of Necessary Data.
- IV Application of Analysis.
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I.

INTRODUCTION- STATEMENT OF THE PROBLEM.

The principal object of the problem proposed in this thesis, is the determination of the amplitude and speed of the oscillations of the pendulum of a shaft governor, when the load on the engine is suddenly changed.

This problem has been treated analytically by Professor Stodola and others. The analytical solution, however, presents many difficulties of application which are avoided in the graphical solution following.



II.

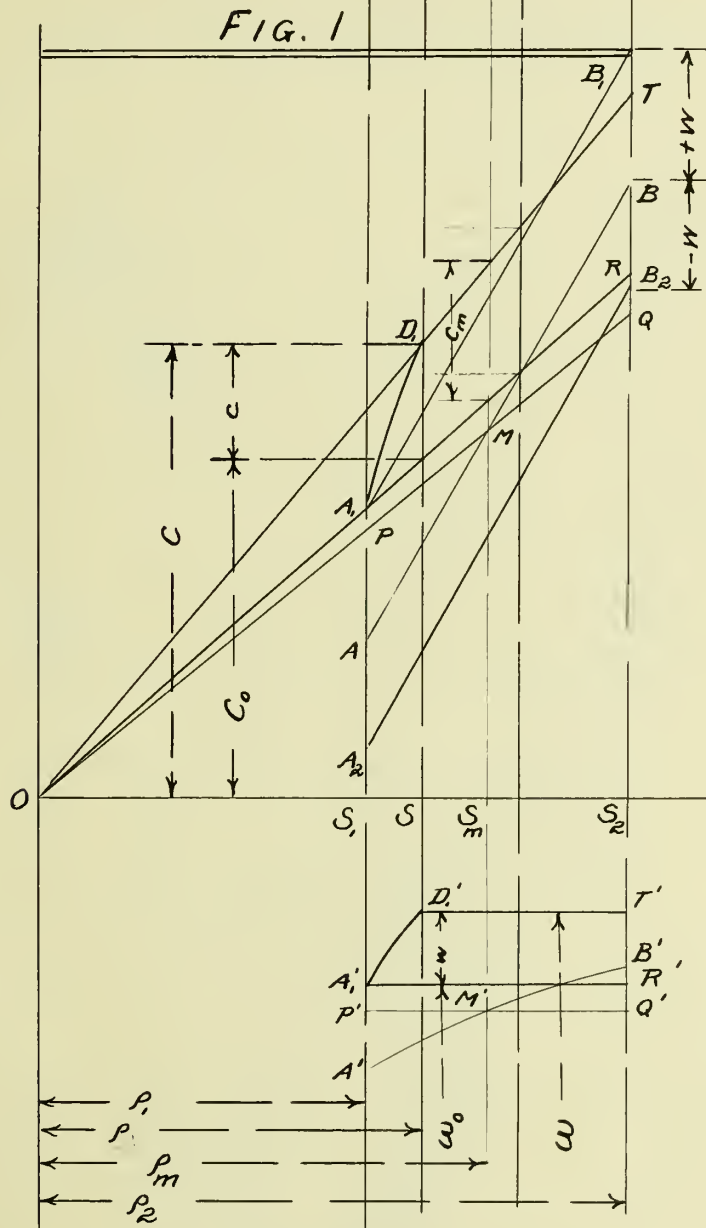
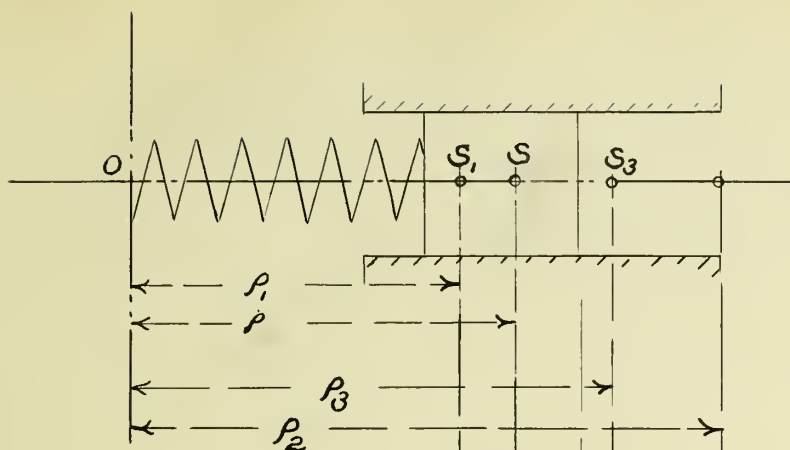
KOOB'S GRAPHICAL SOLUTION OF THE GOVERNOR PROBLEM.

In the "Zeitschrift des Vereines Deutscher Ingenieure" of Feb. 27. 1904, Professor A. Koob, Instructor in the royal technical high school at Munich, published an article entitled, "A Graphical Solution of the Governor Problem."

This analysis applies to all governors in which the governing action is effected by means of centrifugal force. In its application to the shaft governor, the special object is to determine the amplitude and speed of the oscillations of the governor mass when the load on the engine is changed. The following is quoted from Koob's Analysis, with such modifications and omissions as the nature of the governor investigated required.

Equilibrium Line- Astatic Line- Acceleration Curve.

In Fig. 1, O denotes the shaft axis, S the center of inertia of a radially moving mass, and ρ the distance of S from O. S_1 and S_2 are the extreme positions of S. All the masses which give rise to centrifugal force are considered as reduced to the point S and the reduced mass is denoted by μ .



Neglecting gravity the inertia force and spring tension are in equilibrium, and if ω denotes the angular velocity for equilibrium, the inertia force (or centrifugal force) is

$$C = m r \omega^2 \quad (1)$$

Let the value of C that will just equilibrate the spring tension be plotted as ordinates on the path of S as a base line. This line is called the equilibrium line for relative rest. It is shown in AB , Fig. 2.

From (1) the velocities ω for the various values of the centrifugal force shown by AB may be calculated; these plotted give the equilibrium velocity curve $A'B'$.

When the actual motion of the mass is considered certain resistances must be taken into account. The centrifugal force must exceed its value for relative rest by the amount W of this resistance reduced to the point S before outward motion can ensue, and must sink below the value by the same amount before inward motion can take place. Hence the lines A_1B_1 and A_2B_2 at distances W and $-W$ above and below it represent the equilibrium lines for motion.

If the governor mass is shifted from its inner to its outer extreme position, with a constant ω of rotation, the line of centrifugal force will be a straight line passing through the origin. This is shown by equation (1). This line is the astatic line.

A motion of the governor always presupposes a disturbance of equilibrium. If therefore the mass begins to move say from its inner position S_1 , the curve of centrifugal force deviates from the equilibrium line and takes a course as shown by A_1D_1 perhaps. The corresponding curve for ω is $A'_1D'_1$. For the position S the force C is divided into two parts by the astatic line OA_1R and the velocity ω is similarly divided by the corresponding velocity line $A'_1R'_1$. Thus

$$C = C_0 + c \quad \text{and} \quad \omega = \omega_0 + w$$

Applying (1) to the two sides of the equations,

$$C_0 + c = \mu \rho (\omega_0 + w)^2 = \mu \rho \omega_0^2 \left[1 + 2 \frac{w}{\omega_0} + \left(\frac{w}{\omega_0} \right)^2 \right]$$

and $C_0 = \mu \rho \omega_0^2$

Subtracting $c = \mu \rho \omega_0^2 \left[2 \frac{w}{\omega_0} + \left(\frac{w}{\omega_0} \right)^2 \right]$

Since w is always small relative to ω_0 , the last term may be neglected: then

$$\left. \begin{aligned} c &= 2 \mu \rho \omega_0 w \\ w &= \frac{c}{2 \mu \rho \omega_0} \end{aligned} \right\} \quad (2)$$

In Fig. 2 the astatic line OD_1T is drawn through D_1 and in the velocity diagram the corresponding velocity line $D'_1T'_1$ is drawn. Let the segment c be projected centrally from O on the line through S_m , which is chosen at random. Then for the projection C_m , we have

$$\left. \begin{aligned} c_m &= 2\mu\rho_m\omega_0 w \\ w &= \frac{c_m}{2\mu\rho_m\omega_0} \end{aligned} \right\} \quad (3)$$

The part C of the centrifugal force above the astatic line OA_1R may be called the effective motive force or effort; hence we have the following: The velocity fluctuations are proportional to the central projections of the effort on any chosen position of the governor mass.

Equations (2) and (3) completely replace equation (1), and in the following investigation we need consider only the effective effort c and the velocity increment w , not the total force C and velocity ω . This fact renders possible the convenient graphical construction shown in Fig. 3.

Fig. 3 corresponds in all respects with Fig. 2 except that in place of the total quantities C and ω , the deviations from the mean astatic line $OMPQ$ and its image $P'Q'$ are laid off as ordinates. Lines $OMPQ$ and $P'Q'$ of the old diagram become therefore the axes of abscissas. The three equilibrium lines AB , A_1E_1 and A_2B_2 take the positions shown. All astatic lines of course pass through the point O , and the corresponding lines in the velocity diagram are parallel to the axis of abscissas.

Take some value of w in the velocity diagram, say for position S , and lay off as D_1E in the force diagram; and

through E draw a parallel to OD_1 cutting the astatic line OA_1 in G. There is thus obtained a governor position on which the force C is centrally projected from O into a segment whose length represents to scale the velocity increment w corresponding to C. By means of this position UU, which may be called the velocity axis, the velocity curve may be easily derived from the force curve. The position of the velocity axis depends uniquely upon the scale chosen for the velocity diagram.

In Fig. 3 there is still a third diagram shown, indicated by the curve XY, and this brings into consideration the dependence of the effort of the motor on the position of the governor.

It is assumed that any position of the governor, as S, corresponds to a definite effort of the motor. This effort reduced to unit crank radius is denoted by P and the resistance correspondingly reduced by Q. The excess $P-Q$ accelerates the moving masses of the motor, and if M is the moment of inertia of these parts, the acceleration b of the parts for a given position S of the governor is given by the equation

$$b = \frac{P-Q}{M} \quad (4)$$

Laying off the acceleration thus obtained on the governor path as an axis, the curve XY is obtained; it is called the acceleration curve.

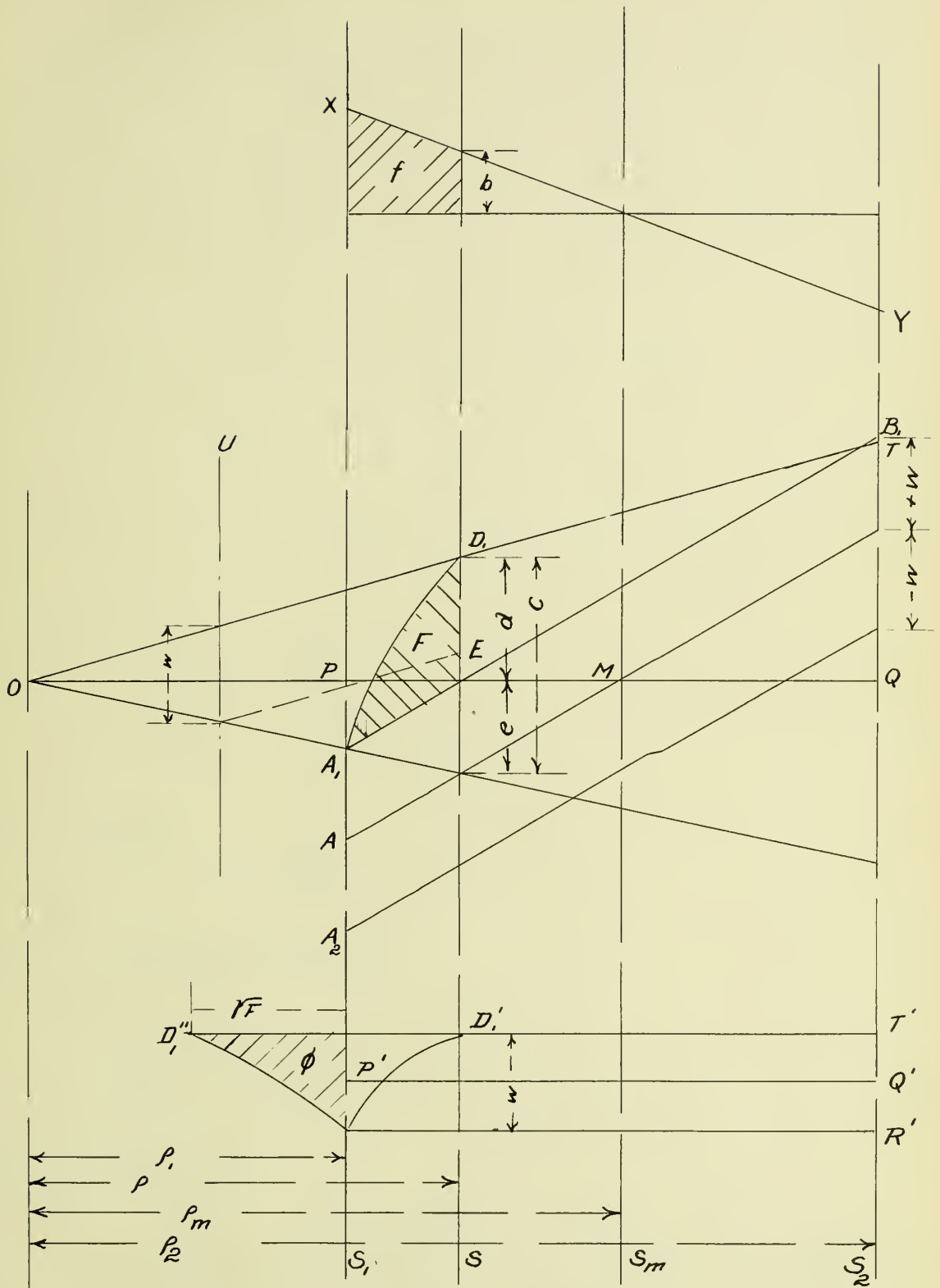


FIG. 3

The Centrifugal Force and Velocity Curves.

Referring to Fig. 3, the motive force c for the governor position S is divided into two parts e and d by the equilibrium line A_1B_1 . Stodola calls the part c the static and the part d the dynamic governing force. The part d is the excess of centrifugal force over that necessary for equilibrium: it acts therefore to accelerate the governor masses. The work done by the force d up to the position S is expressed by $\int_{p_i}^p d \cdot dp$ and is represented by the shaded area F . This work must appear as kinetic energy of the moving masses of the governor. Let M_r denote these masses reduced to the center of inertia of the oscillating mass. Then

$$\left. \begin{aligned} F &= \frac{M_r}{2} \left(\frac{dp}{dt} \right)^2 \\ \sqrt{F} &= \sqrt{\frac{M_r}{2}} \frac{dp}{dt} \end{aligned} \right\} \quad (5)$$

We have further

$$b = \frac{d\omega}{dt}$$

whence

$$b dp = d\omega \frac{dp}{dt}$$

or

$$\frac{dp}{dt} = b \frac{dp}{d\omega}$$

Substituting in (5)

$$\left. \begin{aligned} \sqrt{F} d\omega &= \sqrt{\frac{M_r}{2}} b dp \\ \int_{p_i}^{p_2} \sqrt{F} d\omega &= \sqrt{\frac{M_r}{2}} \int_{p_i}^{p_2} b dp \end{aligned} \right\} \quad (6)$$

Equation (6) admits of a simple geometrical interpretation. If in Fig. 3 we lay off \sqrt{F} as ordinate with corresponding values of ω as abscissas, we get a curve which bounds an area

$$\phi = \int \sqrt{F} d\omega$$

$\int \delta \rho$ is represented by the shaded area f in the acceleration diagram. Equation (6) may therefore be written

$$\phi = \sqrt{\frac{Mr}{2}} f$$

This simple equation is the basis of a method of obtaining the characteristic force and velocity curves for the governor motion. It may be called the fundamental equation of the governor problem.

Let it be assumed that the force and velocity curves have been found as far as points D_1 and D'_1 , Fig. 4 and that the problem is to obtain the additional points E_1 and E'_1 . To find E_1 consider the actual continuous change of the centrifugal force represented by the dotted line to be replaced by a discontinuous change brought about in the following manner: From the position S_1 let the governor mass be suddenly moved into a new position midway between D_1 and E_1 , the ω meantime remaining constant, and let the mass be held there until by reason of the resulting acceleration of the motor parts ω is increased by $\Delta\omega$; then the governor is again suddenly moved (again with ω constant) to the position S_2 . The force curve is

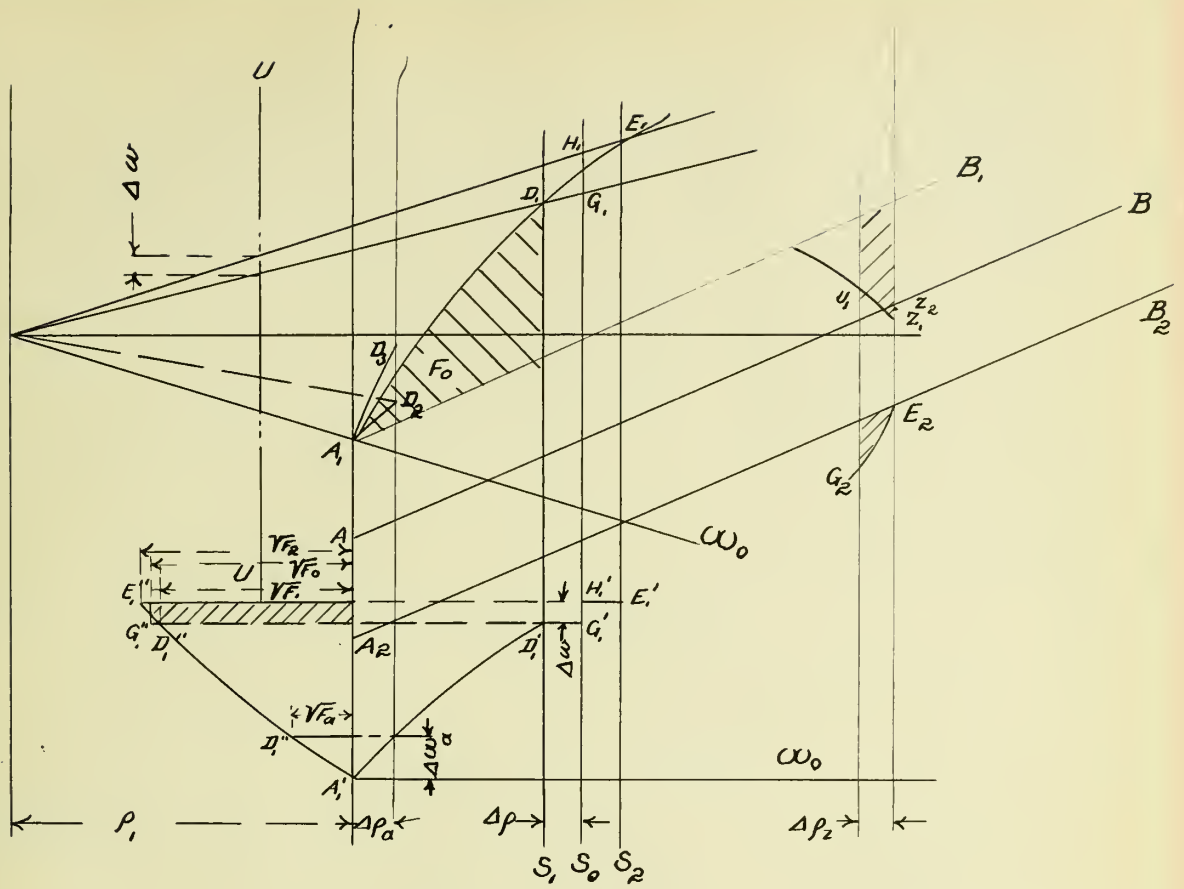


FIG. 4.

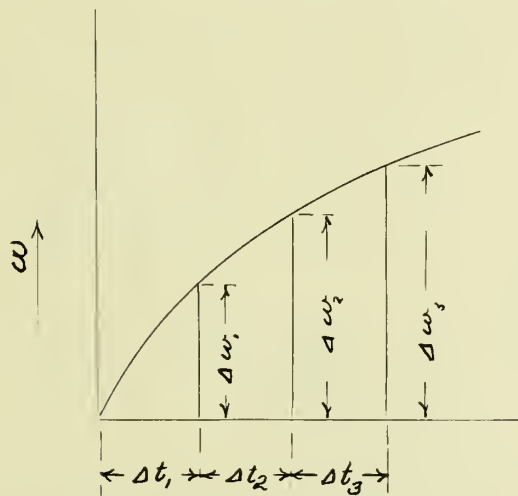


FIG. 5

is thereby replaced by the broken line $D_1 G_1 H_1 E_1$ consisting of two astatic segments $D_1 G_1$ and $H_1 E_1$ and the vertical segment $G_1 H_1$. The velocity diagram has the corresponding broken line $D'_1 G'_1 H'_1 E'_1$. The ordinate of the \sqrt{F} curve, which may have the value $\sqrt{F_1}$ for the position S_1 , increases to $\sqrt{F_0}$ as the governor moves from S_1 to S_0 . It remains constant as the speed increases by $\Delta\omega$ with the governor fixed, and increases further to $\sqrt{F_2}$ when position S_2 is reached.

If now S_0 is taken midway between S_1 and S_2 , the area for the broken line $D_1 G_1 H_1 E_1$ approximates to the area for the actual curve $D_1 E_1$, and in consequence is nearly the true value of the area F for the position S_2 . Approximately therefore the dotted line $D'_1 E'_1$ gives the true course of the \sqrt{F} curve. With sufficient exactness we may take the area under $D'_1 E'_1$ equal to the rectangle, whence

$$\int_{S_1}^{S_2} \sqrt{F} d\omega = \sqrt{F_0} \Delta\omega$$

From (6)
$$\sqrt{F_0} \Delta\omega = \frac{\sqrt{M_r}}{2} \Delta f$$

hence
$$\Delta\omega = \frac{\sqrt{M_r}}{2} \frac{\Delta f}{\sqrt{F_0}}$$

Accordingly the determination of points E_1 and E'_1 proceeds as follows:

The astatic line $D_1 G_1$ is drawn and the area lying under it is added to the known area F_1 . Thus F_0 is found. The

area Δf between the position S_1 and S_2 is easily obtained, and from (10) $\Delta \omega$ can be calculated. $\Delta \omega$ is laid off in the velocity diagram on the S_2 position, thus giving the point E'_1 . Again it is laid off in the force diagram on the velocity axis UU and by central projection from O on the position S_2 , the point E_1 is found. The area under $H_1 E_1$ is added to F_0 , giving the area F_2 for the position S_2 . Successive repetitions of the process give successive points on the desired curves. The method is very simple and the value can be taken relatively large, especially near the middle of the stroke where the force and velocity curves are relatively flat, without causing an appreciable deviation of the curve from its true position.

The method fails however when we begin at point A_1 lying on the equilibrium line, because for this case (10) takes the form $\Delta \omega = \frac{\Delta f}{0}$

Hence the point D_1 must be found in another way.

A point D_2 is assumed on S_a , from (11) a value for F'_a is calculated and then $\Delta \omega'_a$ is calculated from the equation

$$\frac{1}{2} \sqrt{F'_a} \cdot \Delta \omega'_a = \sqrt{\frac{Mr}{2}} \Delta f_a$$

If now $\Delta \omega'_a$ is laid off on the velocity axis UU and projected centrally on the position S_a , a point D_3 is found which will coincide with D_2 in case the assumed position of D_2 is the

correct one. If as in Fig. 4, D_3 falls above D_2 , it follows that in the calculation of $\Delta\omega'_a$ too small a value of $\sqrt{F_a}$ was taken in (13). The correct point D_1 of the force curve therefore lies between D_2 and D_3 . It falls always near the middle point between D_2 and D_3 . It falls always near the middle point between D_2 and D_3 so that this point may be taken as the true one if D_2 and D_3 are not widely separated. Otherwise, the process may be repeated, starting with the middle point D_4 , and D_1 will be obtained midway between D_4 and D_5 .

A similar though simpler procedure gives the point Z_1 of the force curve at the end of the stroke of the governor. Suppose that the curve has been obtained by the method just given as far as the point u_1 and the work area F_u of the dynamic effort that is bounded by the curve is thereby known. The remainder of the force curve can be drawn in with a reasonable degree of accuracy - say it is u_1Z_2 in Fig. 4. The final position S_z is determined by the condition that the total work area of the dynamic effort must reduce to zero, that is $F_z + F_u = 0$

or
$$F_z = F_u$$

if we take account only of the magnitudes of the corresponding areas.

Now if Δp_z is taken small enough we may consider the end element $Y''Z_1'$ of the F -curve (not shown in the figure) as a straight line and again use for our fundamental equation the form of equation (13); thus $\frac{1}{2} \sqrt{F_z} \Delta \omega_z = \sqrt{\frac{Mr}{2}} \Delta x_z$

and
$$\Delta \omega_z = \frac{\sqrt{\frac{Mr}{2}}}{\frac{1}{2} \sqrt{F_z}} \Delta x_z$$

If now we project centrally $\Delta \omega_z$ drawn on the velocity axis on the position S_z we obtain a point Z_1 which in case the tentative curve were correctly drawn must fall on Z_2 . In any case the point Z_1 gives the extremity of the force curve quite accurately; for the actual area F_z exceeds the assumed area by a vanishingly small amount. However, a repetition of the process will correct any slight error. It may be noted that as F_z must have a definite value, the correction will slightly shift the final position S_z in and out, and will thus affect Δx_z .

Since at Z_1 the accumulated kinetic energy of the governor masses has been consumed, the governor is ready to make its return movement. If the resistance is so great that the point Z_1 falls between B_1 and B_2 as in Fig. 5, the governor must remain at rest in the position S_z until the centrifugal force drops to the value E_2 at A_2B_2 . The curve for the return movement then begins at E_2 and it may be constructed by points by

the method just described. First the point G_2 is found in the same way as was used for finding D_2 .

It is possible to continue the drawing of the force curve and its image in the velocity diagram until the oscillations of the governor mass terminate either at the stops or in a position of rest where the driving effort and the resistance of the motor are equal. The curves thus found inform us as to the number of oscillations that will result from a disturbance of proper sequence of events, their amplitude, and the resulting fluctuations in the motor speed. There remains for consideration the time occupied by these occurrences.

Because of the smallness of the intervals into which the path of the governor mass was divided, Fig. 4, in obtaining the points of the force curve it is permissible to take the time in which the mass moves over the space $\Delta \rho$ as equal to the time during which the mass would have to be held fixed in the position S_0 until under the influence of the mean acceleration b , the motor speed would increase by $\Delta \omega$. This time interval is given by $\Delta t = \frac{\Delta \omega}{b}$

and for the spaces $\Delta \rho_\alpha$ and $\Delta \rho_z$

$$\Delta t_\alpha = \frac{\Delta \omega_\alpha}{b_\alpha}$$

$$\Delta t_z = \frac{\Delta \omega_z}{b_z}$$

Laying off these time elements end to end on the axis of abscissas and as ordinates the corresponding velocity fluctuations taken from the velocity curve, we obtain a chronological representation of the velocity as in Fig. 5, which in the following we shall call the time diagram of velocity.

The masses.

It has been shown in the derivation of the fundamental equation of the governor that the moving masses do not all influence the regulation in the same way, but that we must distinguish between the mass m producing centrifugal force and the mass m_r which accumulates kinetic energy. The method of reducing the masses to a single point, which is always taken as the center of inertia of the principal oscillatory mass, will depend upon the geometry of the system.

We need only to keep in mind that in accordance with the significations attached to the symbols m and m_r in the preceding investigations,

All centrifugal force producing masses are to be replaced by a single mass m at the reduced point whose centrifugal force is the resultant of all the single centrifugal forces, and all moving masses of the governor and of valve gear attached to it are to be replaced by a mass m_r which with a velocity $\frac{dp}{dt}$ possesses the same kinetic energy as all the single masses.

Inertia Action.

The acceleration or retardation of the rotating masses of a governor due to the change in the angular velocity gives rise to inertia forces. In the investigation of the governor problem these forces, generally speaking are to be taken into consideration if they are able to assist or hinder the governor motion, that is if they exert a turning moment on the essential organ of the governor, the ball or the pendulum, with respect to the axis of rotation.

The significance of the inertia action as affecting the regulation will now be shown.

In Fig. 6 the governor pendulum is shown (by center line only) in the position DS. A small motion of the governor about the shaft center O, brings the pendulum to the dotted position D'S'. The point D moves to D' and the motion of the pendulum may be replaced by a translation perpendicular to the radius whereby D is moved to D'' followed by a rotation of the pendulum about its center of inertia S', whereby D moves along the path D''D' perpendicular to DS. The triangle DD'D'' and ODS are similar, so that

$$\frac{D'D''}{DD''} = \frac{R}{\rho}$$

The angular acceleration b may be replaced by an acceleration of translation $DD'' = \rho b$ and an angular acceleration about S of

$$\text{magnitude } \frac{D'D''}{R} = \frac{DD''}{R} \frac{R}{\rho} = \frac{\rho b}{R} \cdot \frac{R}{\rho} = b$$

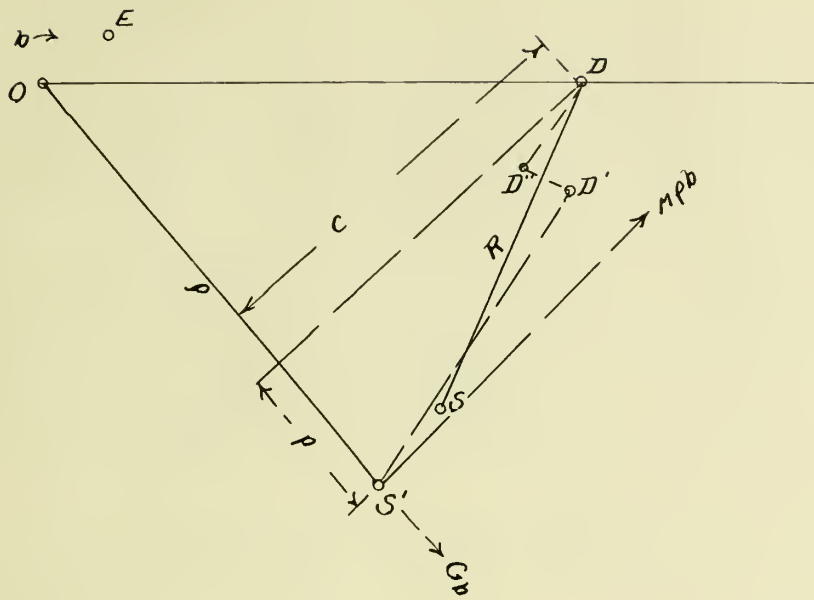
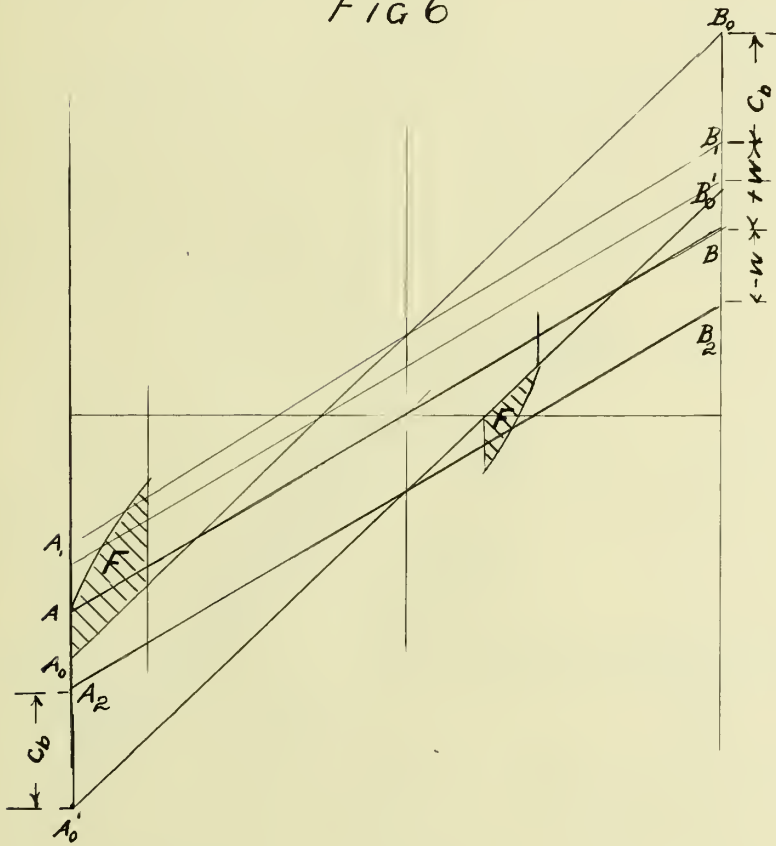


FIG 6



The first component gives rise to parallel inertia forces in the mass element of the pendulum whose resultant passes through S and is perpendicular to the radius ρ . If μ is the total mass of the pendulum, the magnitude of this resultant is $\mu \rho \delta$, and its moment about D is $\mu \rho \delta \rho$

The angular acceleration b about S gives rise to an inertia couple whose moment is

$$I \delta$$

where I denotes the moment of inertia of the pendulum relative to the center of inertia S. The whole inertia action of the pendulum consists therefore of a torque about the center of rotation D of magnitude

$$M = (I + \mu \rho^2) \delta \quad (38)$$

The moment may be replaced by a single force C_b through S having a line of action OS and a magnitude given by

$$C_b c = (I + \mu \rho^2) \delta$$

According to Stodola C_b may be called the inertia force and $C_b c$, the inertia moment.

In the present case the inertia moment assists the governing motion of the pendulum. The equilibrium centrifugal forces as determined by the moment FF of the spring tension

are drawn in Fig. 6 and give the equilibrium line AB. During the motion the point S of the pendulum is acted upon not only by the excess d of the centrifugal force over the equilibrium centrifugal force and by the force of friction W , but also by the inertia force C_p ; hence the regulation is the same as if A_0B_0 were the equilibrium line for centrifugal forces. The inertia action therefore is equivalent to an increase in the stability of the governor. The method of finding the characteristic curves of such a governor has the same features as that previously described, only the free governing forces and their energy areas F must be measured from the line A_0B_0 .

The Regulation of the Steam Engine.

With a governor which regulates the admission of a steam engine, the characteristic curves differ somewhat from those of a governor with uninterrupted action, and the method of obtaining these curves differs somewhat from the method developed in the first part. In this connection see Fig. 7.

Consider a steam engine at the instant of cut-off, and suppose it to be unloaded at this instant. Previously a state of equilibrium existed, and after the unloading the effort and resistance will first be equal to each other for the governor position S_m . At the instant of unloading the excess of effort for position S_1 gives rise to an acceleration of the motor shaft

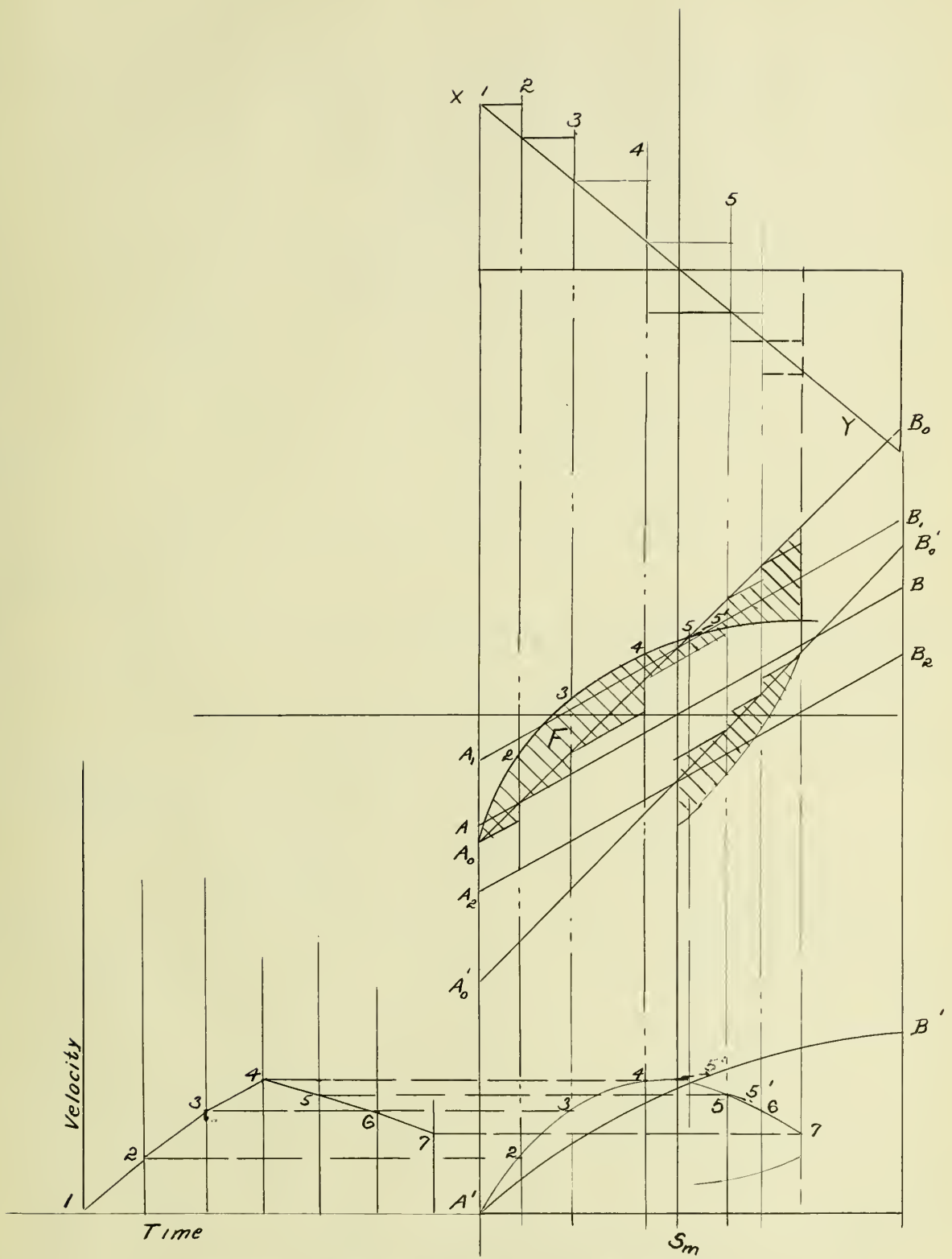


FIG. 7

and governor shaft, which up to the next cut-off is not under control of the governor, and which varies constantly because of the periodical variation of the tangential effort peculiar to the steam engine. For the sake of simplicity of treatment, the varying acceleration up to the next cut off may be replaced by a constant mean acceleration, which will be denoted by b_0 . Under the action of b_0 , the governor begins to move towards the equilibrium position S_m as soon as the centrifugal force increases by the amount of the resistance W . The centrifugal force and velocity increase to the next cut off from point 1 to point 2, Fig. 7 .

From the assumption of constant acceleration, the increase of velocity is proportional to the time, and in the velocity diagram on a time base, the change of velocity is represented by the straight line 1 - 2.

At point 2 the next period of regulation begins. The mean value of the acceleration is smaller, corresponding to the new position 2 of the governor. The acceleration diagram appears as a series of steps as shown in the figure: since the acceleration is assumed constant between cut offs, the inertia force may be taken as constant; the centrifugal force and velocity curves appear as broken lines; and the velocity diagram on time base is a series of rectilinear segments (1 - 2), (2 - 3), etc., the segments on the time base representing the

time intervals from cut off to cut off.

The accelerations that would arise for the different governor positions, with the governor held fixed are represented as ordinates of a curve which is assumed to be the dash line XY. These correspond to the mean efforts that would be exerted in a state of equilibrium. This line may be found accurately for any given case.

The time from one cut off to the next varies, first because the speed of the motor varies, and secondly because for the same piston position the valve gear may have different positions in successive strokes. It is exact enough, however, to consider this time element t_0 constant and calculate it from the mean motor speed. Thus for single acting engines

$$t_0 = \frac{60}{N_m} \quad (40)$$

and for double acting engines

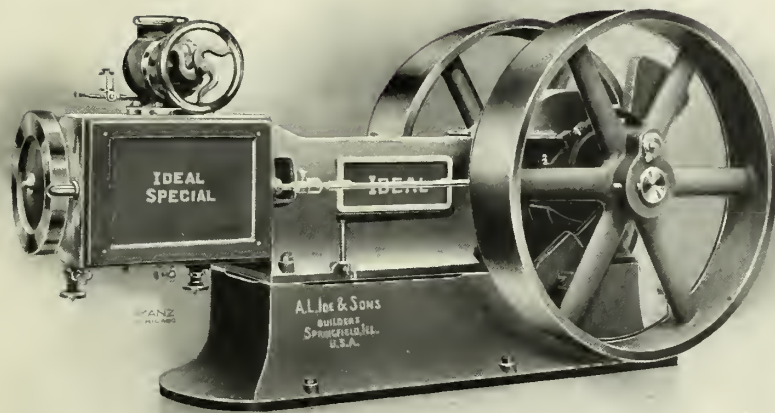
$$t_0 = \frac{30}{N_m} \quad (40a)$$

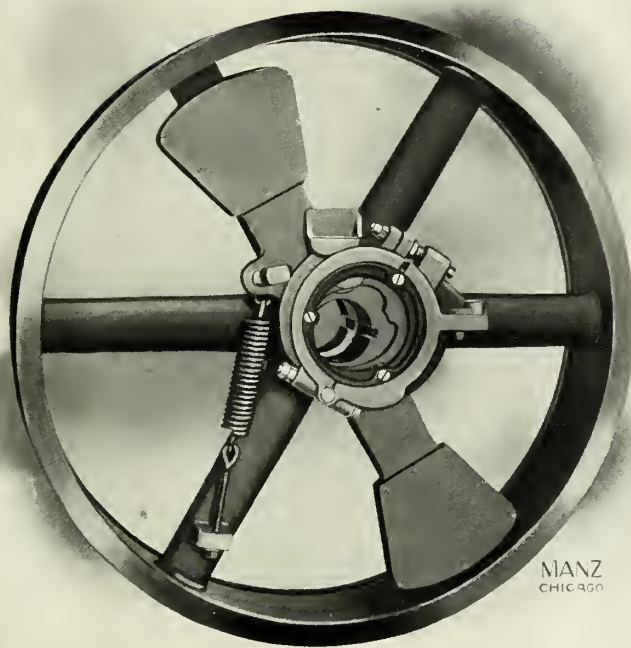
If now the governor curves were determined as far as point 4, the following point 5 in the time diagram may be indicated by calculating the increase of velocity $\Delta \omega_4$ from the equation $\Delta \omega_4 = \delta_4 t_0$

$$(41)$$

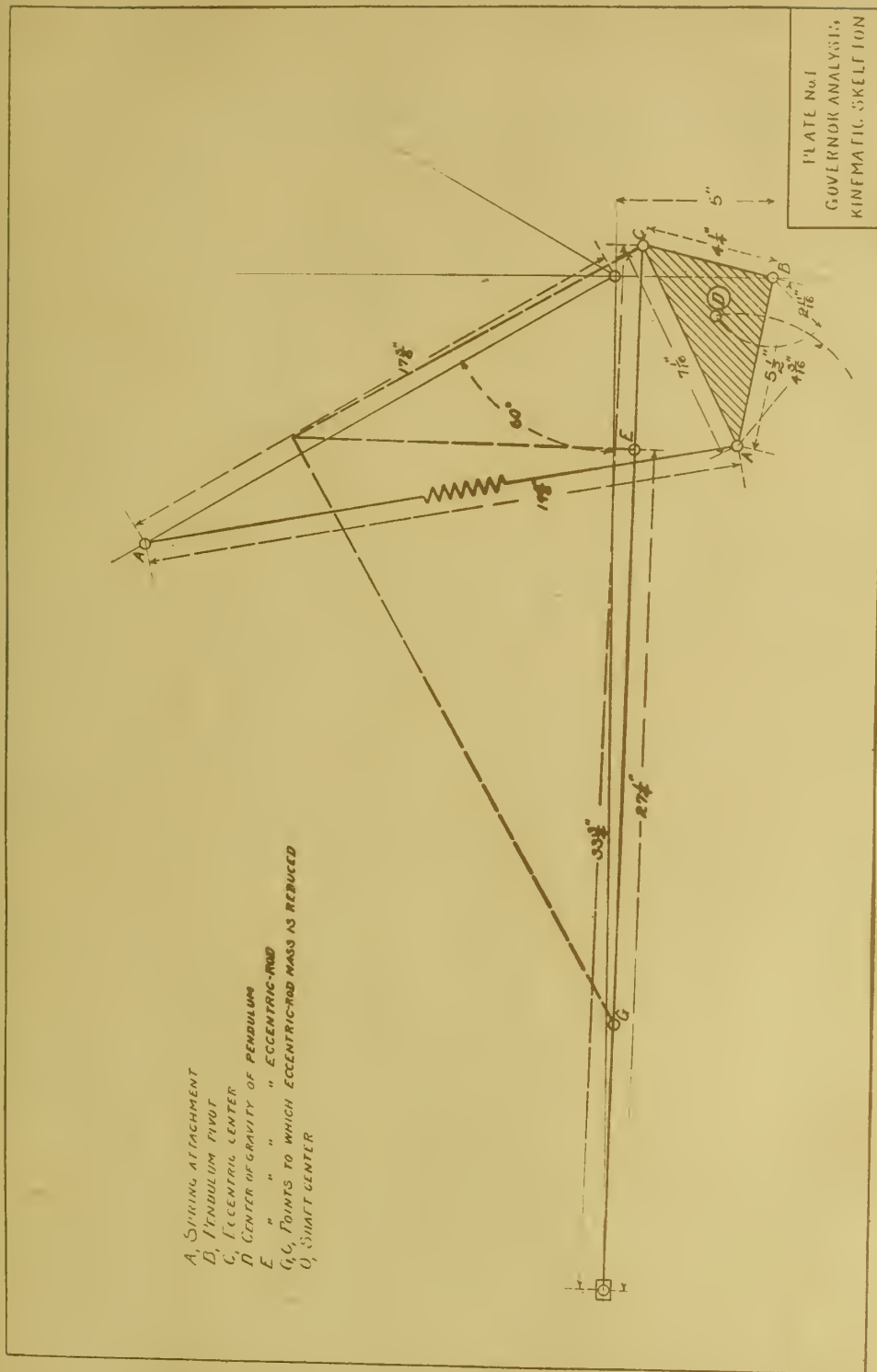
On the other hand it is not possible to fix immediately the point 5 in the other diagrams because it is not known how far the governor will have moved in t_0 seconds. We may,

however, choose arbitrarily a width 45' and by the usual method obtain the velocity and force curves. Then from the time diagram we project the point 5; the projector cuts the assured or tentative curve 4 - 5 in the point 5 and determines thus the position S of the governor at the beginning of the next period of regulation. Laying off this position S in the force and acceleration diagrams we get in the former the point 5 of the force curve and in the latter the acceleration (-b) for the next period, which is negative. Proceeding in this way from the beginning of the regulation, the characteristic curves for such a case may be obtained with but little extra labor.





MANZ
CHICAGO



III.

DETERMINATION OF NECESSARY DATA.

The governor chosen for investigation was a Rites inertia governor, used on a 10" x 10" Ideal engine in the steam laboratory of the University of Illinois. Cuts of the engine and governor are shown on page . The R.P.M. were given as 300. The slide valve was of the flat, balanced type. Plate I shows the kinematic arrangement of the governor and its connection to the slide valve.

The following is the data that had to be obtained in the laboratory:

- (1) The moment of inertia of the rotating parts of the motor - the fly wheels, crank discs, shaft, and governor mass.
- (2) The moment of inertia and center of gravity of the pendulum and of the eccentric-rod and strap.
- (3) The mass of the valve and valve-rod.
- (4) The friction of the pendulum on its pivot, the friction in the eccentric strap, and in the slide valve and stuffing box.
- (5) The net torque that gets to the crank pin, at constant steam pressure, for various positions of the pendulum.

(6) The relation between pull and extension of governor spring.

(7) The dimensions for the kinematic skeleton of the governor mechanism and valve gear.

To obtain the moment of inertia of the flywheels, one of them was removed, suspended, and caused to oscillate. The distance of the center of gravity from the point of suspension was noted, also the time of oscillation. From this data the moment of inertia was calculated. The moment of inertia of the shaft and crank-discs was small and was calculated from measurements and estimates. The moment of inertia of the pendulum and of the combined eccentric-rod and strap was obtained in the same way as that of the fly wheel. Its center of gravity was determined by balancing them in two different positions on a knife edge and noting the point of intersection of the lines of contact. The valve and valve rod were taken out and weighed to determine their combined mass.

The net torque exerted by the engine for various governor positions was obtained as follows: The rubber stop for the pendulum was removed and an adjustable stop put in its place. The pendulum was held rigidly against this stop by tightening the governor spring. A prony brake was placed on the fly wheel, and indicators were attached to the cylinder. Steam was admitted to the cylinder at full pressure. The

brake load was adjusted until the engine ran at same constant speed, approximately the rated speed of the engine. Indicator cards were also taken as a check on the brake readings. Readings were taken for several different governor positions. The steam pressure was nearly constant while the readings were being taken.

The relation between pull and extension of the governor spring was determined by means of a testing machine and extensometer.

The data taken in the laboratory is shown on pp. 42, 43.

IV.

APPLICATION OF ANALYSIS.

Mass Reductions.

In the Rites inertia governor, Plate I, the masses giving rise to centrifugal force are: the mass of the pendulum, which may be considered concentrated at its center of gravity, and that part of the mass of the eccentric rod and strap that may be considered concentrated at the eccentric center. These two masses may be replaced by one which has the same mass and center of gravity as their sum. The centrifugal force required for equilibrium in various governor positions is obtained by making the force such that its moment about the pendulum pivot is equal to the moment of the spring tension. It is obtained graphically in Plate II.

The energy storing masses are

M_1 mass of the pendulum.

M_2 " " " eccentric rod and strap.

M_3 " " " valve rod and valve.

Let m_1 denote an element of M_1 at a distance x from pivot.

m_2 " " " " M_2 " " " y " valve-rod joint (B)

Let m_3 denote an element of M_3

m_1' " the mass element m_1 reduced to S

m_2' " " " " m_2 " " eccentric center E

M_1' " reduced mass of pendulum.

M_2' " " " " eccentric rod.

M_3' " " " " valve rod and valve.

I_1 = moment of inertia of pendulum relative to S

I_2 = " " " " eccentric rod relative to B

θ = angle between crank and line of stroke.

$\frac{d\phi}{dt}$ = angular velocity of pendulum relative to fly wheel.

R = distance from pivot to center of gravity of pendulum.

The speed of m_1 is $x \frac{d\phi}{dt}$, and the speed of the reduced masses at the center of gravity of pendulum is $R \frac{d\phi}{dt}$.

Hence
$$\frac{m_1}{2} x^2 \left(\frac{d\phi}{dt} \right)^2 = \frac{m_1'}{2} R^2 \left(\frac{d\phi}{dt} \right)^2$$

or
$$m_1 x^2 = m_1' R^2$$

$$\sum m_i x^2 = \sum m_i' R^2 = R^2 \sum m_i'$$

whence
$$I_1 = M_1' R^2$$

or
$$M_1' = \frac{I_1}{R^2}$$

The speed of the eccentric center relatively to the wheel is $R \frac{d\phi}{dt}$, and this may be resolved into two components $R \frac{d\phi}{dt} \sin \theta$ and $R \frac{d\phi}{dt} \cos \theta$. With sufficient accuracy we may re-

place the motion of the element m_2 by an oscillation about the end (B) with the velocity $y \frac{l}{l_1} \frac{d\varphi}{dt} \cos \theta$ and a velocity at right angles of magnitude $l \frac{d\varphi}{dt} \sin \theta$. Hence the relation

$$\frac{m_2'}{2} R^2 \left(\frac{d\varphi}{dt} \right)^2 = \frac{m_2}{2} \left[y^2 \left(\frac{l}{l_1} \right)^2 \cos^2 \theta + l^2 \sin^2 \theta \right] \left(\frac{d\varphi}{dt} \right)^2$$

$$\sum \frac{m_2'}{2} R^2 = \sum \left(m_2 y^2 \left(\frac{l}{l_1} \right)^2 \cos^2 \theta + \sum m_2 l^2 \sin^2 \theta \right)$$

$$\sum m_2 y^2 = I_2 = \text{moment of inertia of the eccentric.}$$

$$\sum m_2 l^2 = M_2 l^2$$

Since θ varies from 0 to 2π during the revolution, we may take mean values of $\cos^2 \theta$ and $\sin^2 \theta$ as follows:

$$(\cos^2 \theta)_m = (\sin^2 \theta)_m = .5$$

$$\sum m_2' R^2 = M_2' R^2$$

$$\text{Hence } M_2' = \frac{.5 l^2}{R^2} \left[\frac{I_2}{l^2} + M_2 \right]$$

The velocity of the valve and rod is approximately $l \frac{d\varphi}{dt} \sin \theta$, so that

$$m_3' R^2 \left(\frac{d\varphi}{dt} \right)^2 = m_3 l^2 \left(\frac{d\varphi}{dt} \right)^2 \sin^2 \theta$$

Hence, as before, taking a mean value of $\sin^2 \theta$

$$M_3' = .5 \frac{l^2}{R^2} M_3$$

The total mass reduced to the point S the center of gravity of the pendulum is $M' = M_1' + M_2' + M_3' =$

$$\frac{I_1}{R^2} + .5 \frac{L^2}{R^2} \left[\frac{I_2}{L^2} + M_2 + M_3 \right]$$

The value of M' is calculated on page

The reduced mass M_r which has the velocity

$$\frac{dp}{dt} = R \frac{d\phi}{dt} \cos \alpha$$

is given by the equation

$$M_r = \frac{M'}{\cos^2 \alpha}$$

and may be obtained by the graphical construction shown in Plate II.

Effect of Friction Forces.

Three principal frictional forces of the governor and valve gear are the friction in the pendulum pivot, the friction in the eccentric strap, and the friction in the slide valve and stuffing box.

The friction in the pendulum pivot always opposes the motion of the pendulum relative to the fly wheel, hence has a greater influence on the motion of the pendulum than any other frictional force. Its moment is determined as follows. Neglecting the friction of the eccentric strap and the acceleration of the governor-mass, the pendulum is in equilibrium in

any position under the influence of the spring tension, the centripetal force reversed and the pin pressure. The first two can be calculated, and the pin pressure found from the force triangle. From the pivot pressure, the diameter of the pivot, and an assumed coefficient of friction, the moment of friction can be found.

The friction of the slide valve alternately assists and opposes the action of the spring pull on the pendulum. On this account, and as its magnitude is considerably less than that of the pivot friction, it has been neglected in this problem. Its effect would probably be to increase slightly the speed, amplitude, and number of the pendulum oscillations.

The friction of the eccentric strap always opposes the spring tension. Its effect is simply to lessen the centrifugal force required to equilibrate the spring tension. The pressure in the strap is made up of three components that due to the pull on the valve rod, that due to its own weight, and the centrifugal force of that part of the mass of the strap and rod considered concentrated at the eccentric center. As this last force is the greatest, and is constantly changing its direction the effect of the other two forces may be neglected. The method of calculating the moment of friction is the same as for the pivot.

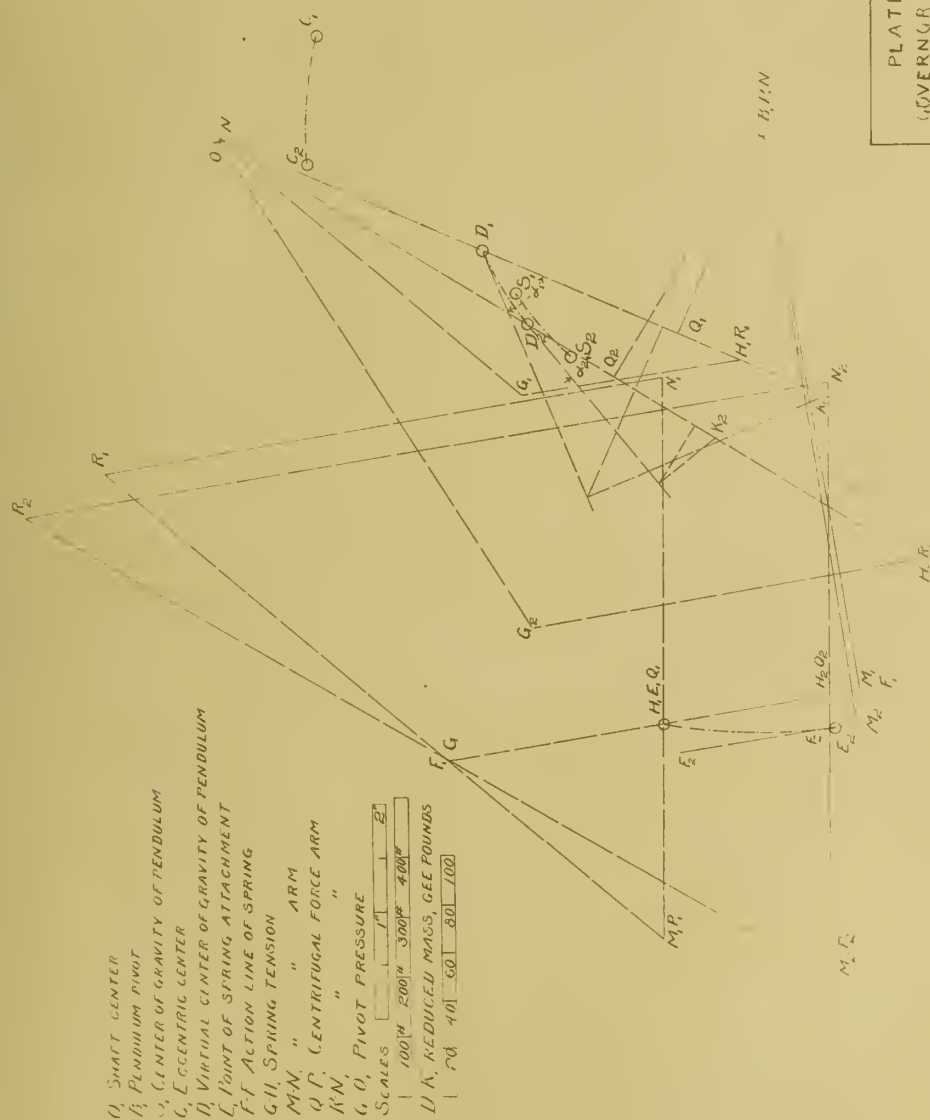


PLATE No. II
GOVERNOR ANALYSIS
FORCE & MASS REDUCTION

V.

CALCULATIONS.

Moment of Inertia.

The moment of inertia of the different parts were calculated from the formulae, $t = \pi \sqrt{\frac{l'}{g}}$, $l' = h + \frac{k^2}{h}$, $I = mk^2$, in which t is the observed time of oscillation, l' is the length of the simple equivalent pendulum, g the acceleration of gravity, h the distance from center of suspension to center of gravity, k the radius of gyration, m the mass, and I the moment of inertia. The data and calculations are shown on page 42.

Motor Acceleration.

The motor acceleration corresponding to various positions of the pendulum was obtained from the formula, $P = I\alpha$, in which P is the net torque, obtained from the brake pull, I the moment of inertia of the rotating parts, and α the resulting motor acceleration.

Angular Velocity.

The centrifugal force for equilibrium is found graphi-

cally in Plate II. The corresponding angular velocity is calculated from the formula, $\omega = \sqrt{\frac{C}{Mp}}$ in which ω is the angular velocity, C the centrifugal force for equilibrium, M the mass giving rise to centrifugal force, and p the distance from the center of gravity of the centrifugal force producing mass to the shaft center.

Reduced Inertia Forces.

It has been shown that the reduced inertia force is

$$C_d = (I \pm Mp^2) \frac{z}{c}$$

in which C_d is the inertia force reduced to the center of gravity of the centrifugal force producing mass, I the moment of inertia of this mass about its center of gravity, M its mass, p the distance from its center of gravity to the shaft center, c the perpendicular distance from the pendulum pivot to a line through the center of gravity of the centrifugal force producing mass perpendicular to p , z the moment arm of the centrifugal force about the pendulum pivot, and b the difference between the positive acceleration of the motor and the negative acceleration of the load. The data and calculations are shown on page 46.

Reduced Mass Producing Centrifugal Force.

As has been stated, the masses producing centrifugal force are, the pendulum, and that part of the eccentric strap and rod that may be considered concentrated at the eccentric center. The mass of the pendulum was found to be 4.08. The mass of the part of the eccentric strap and rod was found to be .77. The total centrifugal force producing mass is 4.85.

The moment of inertia of the centrifugal force producing mass was determined as follows: The distance from the center of gravity of the pendulum to the center of gravity of the centrifugal force producing mass was found to be .041 ft. The distance from the center of gravity of the centrifugal force producing mass to the eccentric center was found to be .166 ft. Therefore the moment of inertia of the two masses about the center of gravity of the centrifugal force producing mass is

$$4.08 + 4.08 \times .041^2 + .77 \times .166^2 = 4.10$$

Reduced Energy -storing Mass.

$$M' = M_1' + M_2' + M_3' = \frac{I}{R^2} + .5 \frac{1^2}{R^2} \left[\frac{I_1^2}{1^2} + M_2 + M_3 \right]$$

$$I_1 = \bar{I}_1 + M_1 R^2$$

$$\bar{I}_1 = 4.06$$

$$M_1 = 408$$

$$R = .229$$

$$I_1 = 4.06 + 4.06 \times .229^2 = 4.266$$

$$l = .354'$$

$$l_1 = 2.83'$$

$$I_2 = \bar{I}_2 + M_2 a_2^2$$

$$\bar{I}_2 = 1.10$$

$$M_2 = 1.07$$

$$a_2 = 2.29'$$

$$I_2 = 1.10 + 1.07 \times 2.29^2 = 6.75$$

$$M_3 = 1.02$$

$$I' = \frac{4.266}{.229^2} + .5 \frac{.354^2}{.229^2} \left[\frac{6.75}{2.83^2} + 1.07 + 1.02 \right] = 84.5$$

$$I_r = \frac{I'}{\cos^2 \alpha}, \text{ and is obtained graphically (Plate II)}$$

Calculation of Mass and Moment of Inertia.

Part	Weight lbs.	Time sec.	Number of oscil- lations	h ft.	l' ft.	$l'h$	h^2	k^2	M	Mk^2
Fly wheel	1232	300	284	2.04	3.67	7.5	4.15	3.35	38.4	128.
Pend- ulum #	136.5	240	281	1.87	2.39	4.48	3.52	.96	4.24	4.06
Con- nect- ing rod	34.5	180	250	.91	1.70	1.87	.84	1.03	1.07	1.10
Valve and rod	33	-	-	-	-	-	-	-	1.03	-
Shaft	-	-	-	-	-	-	-	-	-	.14*
Crank	-	-	-	-	-	-	-	-	-	2.66*
Clamp	5	60	150	.167	.525	.0875	.028	.06	.156	.0095
Pend- ulum	131.5	-	-	-	-	-	-	-	4.03	4.05
2nd fly wheel	-	-	-	-	-	-	-	-	-	128.
Rota- ting parts	-	-	-	-	-	-	-	-	-	263.

* calculated from measurements.
includes clamp.

Calculation of Torque from Cards.

Position	M. E. P.		Work per Rev.		Torque	Distance between lugs
	Head	Crank	Head	Crank		
1	--	--	--	--	--	--
2	32.5	30.9	2550	2320	646	5 9/32"
3	43.2	41.1	3390	3080	855	4 21/32"
4	49.4	45.4	3870	3410	968	3 7/8"
5	59.7	54.0	4680	4050	1160	3 7/16"
6	68.8	61.3	5390	4590	1328	3"
7	78.0	69.8	6120	5240	1512	2 29/64"

Calculation of Motor Acceleration
for various Governor Positions from Brake Pull.

Posi- tion	Distance between lugs inches	Total Pull lbs.	Net Pull lbs.	Net Torque lb.ft.	Motor Acceler- tion rad/sec.
1	5 $\frac{25}{32}$	160	60	315	1.20
2	5 $\frac{9}{32}$	208	108	568	2.16
3	4 $\frac{21}{32}$	245	145	762	2.89
4	3 $\frac{7}{8}$	273	173	910	3.46
5	3 $\frac{7}{16}$	303	203	1065	4.05
6	3	340	240	1260	4.78
7	2 $\frac{29}{32}$	371	271	1420	5.40

Calculation of Angular Velocity.

C		
pounds	feet	rad./sec.
940	.222	29.6
976	.228	29.9
1025	.234	30.1
1072	.241	30.4
1112	.246	30.7
1165	.253	30.9
1212	.259	31.1
1252	.265	31.3
1304	.271	31.5
1350	.277	31.8

$$M = 4.85$$

$$c = M^2$$

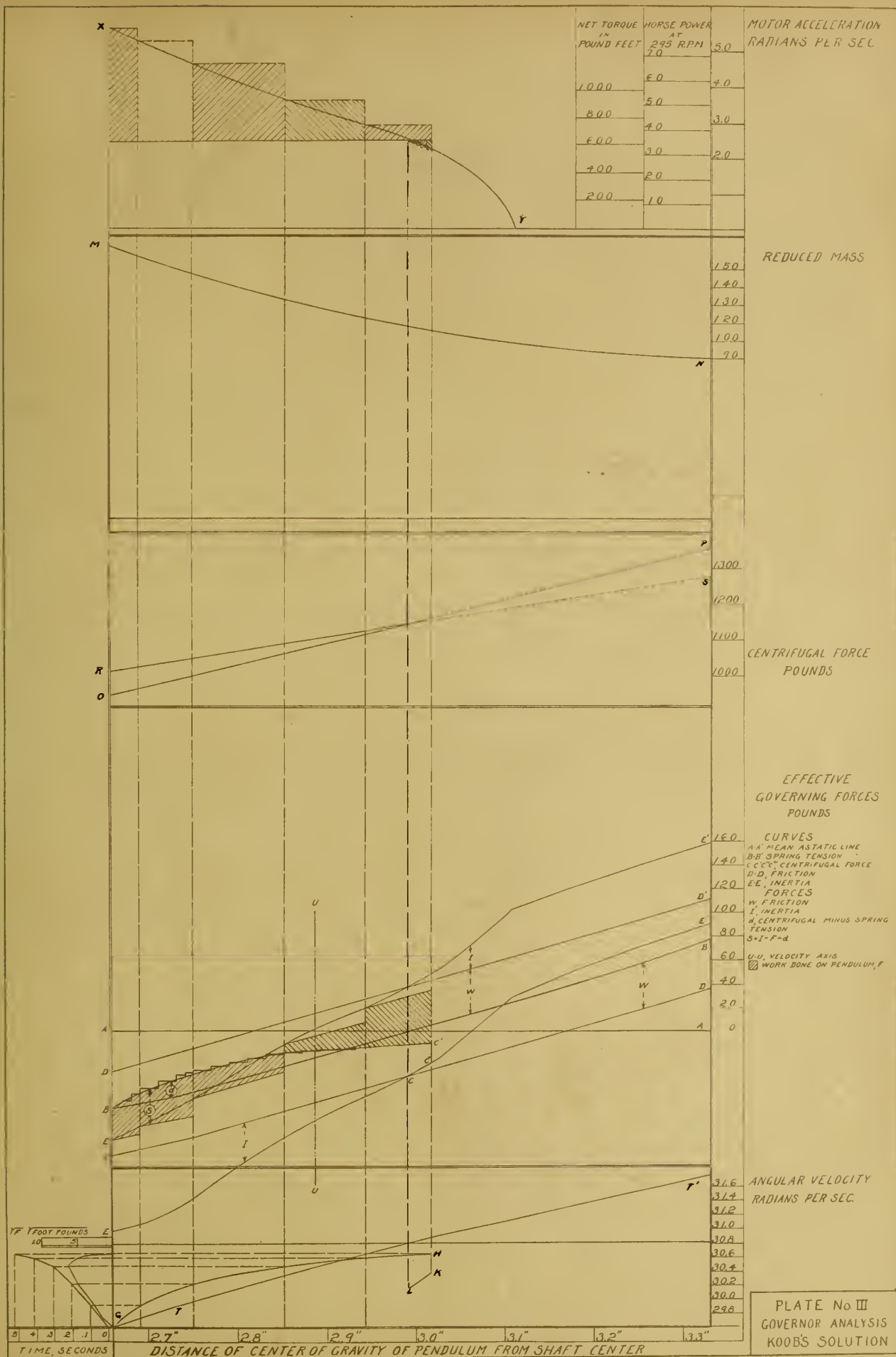
$$\omega = \sqrt{\frac{c}{M \rho}}$$

Calculation of Reduced Inertia Forces.

b	p'		M p	c'	$(I - M p) \frac{b}{c}$
3.05	.160	.222	.174	.167	71.70
2.21	.149	.228	.165	.173	50.80
1.50	.141	.234	.160	.184	32.10
.73	.131	.240	.152	.191	15.10
.30	.122	.246	.146	.197	6.04
- .46	.112	.253	.137	.203	- 9.00
-2.05	.101	.260	.126	.209	-39.00
-2.50	.093	.265	.119	.212	-47.00
-2.50	.083	.271	.110	.215	-46.50
-2.50	.078	.278	.105	.217	-46.00

$$I = 4.10$$

$$M = 4.85$$



In Plate No. III, the centrifugal force and velocity curves are determined from the data obtained. XY shows the motor acceleration that corresponds to any governor position, as determined on page 44. MN shows the reduced energy storing mass for any governor position, as determined in Plate II. OP shows the centrifugal force necessary to equilibrate the spring tension, as determined in Plate IV. RS is the mean astatic line. BB represents the spring tension line laid off from the mean astatic line AA. Lines DD' are laid off from BB at a distance W, equal to the reduced friction force, as determined in Plate II. Lines EE' are laid off from DD' at a distance I equal to the reduced inertia force as determined on page 46. TT' shows the angular velocity corresponding to BB as determined on page 45. BC'C"C shows the path of the centrifugal force curve. GHKL is the corresponding angular velocity curve.

VI

CONCLUSIONS.

According to the analysis when the load on the engine is suddenly changed from 30 H.P. to 35 H.P., the pendulum will make only two oscillations.

The first oscillation occupies .52 seconds. Its amplitude is $7^{\circ} 30'$. During this time there are five cut-offs. The angular velocity increases from 29.6 to 30.6 radians per second, (284 - 292 R.P.M.).

After the first oscillation, the pendulum remains at rest 7.9 seconds. During this time there are 77 cut-offs. The angular velocity decreases from 30.6 to 30.4 radians per second, (292 - 290 R.P.M.)

The second oscillation occupies .50 seconds. Its amplitude is $50'$. During this time there are 5 cut-offs. The angular velocity decreases from 30.4 to 30.2 radians per second, (290 - 288 R.P.M.). The governor comes to rest in a position at which the motor effort is equal to the load.

When the load on the engine is increased from zero to full load (40) H.P. the speed changes from 298 to 292 R.P.M. This gives a regulation of 2 %.

The analysis shows that by tightening the governor spring or adjusting the governor mass, the governor could be made to regulate much more closely without seriously impairing its stability.





